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DYNAMIC CHARACTERISTICS OF RESONATORS UNDER THE ACTION OF VARYING FREQUENCY

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ABSTRACT

The transient oscillation theory has been a subject of numerous investigations in the field of not only radio engineering (A.Kharkevich, D.Fedotov, I.Turbovich, S.Khlytchiev and others), but also in mechanics (A.Kats, Ye.Goloskokov, A.Filippov, A.Lenk, H.Hage, H.Helm, V.Nazarenko [1-6] and others). The results obtained in radio engineering are also recommended for applying to the problems of vibration tests of mechanical systems. Here no strict restrictions are imposed on the field of application of these results. The results of this paper can be used for designing the spectrum analyzers to be calculated based on the dynamic AFC parameters at arbitrarily high excitation frequency scanning rate, if the rate is constant.

1. INTRODUCTION

The analysis of the solution of the problem of transition of a linear system through the resonance has shown the following. The mathematical formulation of the problem in radio engineering coincides with the problem in mechanics. The exact values of the complex transfer constant (ratio of the amplitude of the output action to that of the input one) are obtained by calculating the integral

$$K_T = \int_0^t g(\tau) e^{-i\omega\tau} e^{i\nu_\omega \tau^2/2} d\tau, \qquad (1)$$

where $g(\tau)$ is the impulsive reaction of the resonator, ω is the instant circular frequency, ν_ω is the rate of its scanning.

All the points of the dynamic amplitude-frequency characteristic (AFC) can be obtained depending on the frequency scanning rate for any resonator. In practice, the following parameters determining the dynamic characteristic shall be obtained:

1. Displacement of the maximum of the dynamic characteristic relatively to the maximum of the stationary AFC.
2. Measurement of the maximum value of the dynamic magnification factor.
3. Expansion of the dynamic characteristic relatively to the stationary one.
4. Displacement of the dynamic characteristic relatively to the stationary one.

2. THE MATHEMATICAL MODELS OF THE RESONATORS

The matters of calculation and construction of the spectrum analyzers comprising electronic resonators and using the frequency-modulated testing voltage coincide with the problem of accelerated tests of products under the transient oscillation conditions because they require the definition of the same parameters of the dynamic characteristic. Therefore, the solution of the integral (1) is also of interest from the mechanical point of view. To make it simpler, the upper limit shall be substituted by the infinity, the rate shall be assumed to be $\nu_\omega \to 0$ and the known expression for the static transfer factor $\boldsymbol{K}$ shall be obtained as a Fourier transform of the impulsive reaction. The difference between the transform factors shall be obtained as follows

$$\Delta K = K_T - K = \int_0^\infty g(\tau) e^{-i\omega\tau} (e^{i\nu_\omega \tau^2/2} - 1) d\tau. \qquad (2)$$

On the assumption that the argument $\nu_\omega \tau^2$ to be small for all the τ values, for which the descending $g(\tau)$ has still noticeable value, the expression in parentheses under the integral sign (2) shall be expanded in a power series and restricted to the two members of the expansion. Then the formulae for determining the parameters of the dynamic characteristic of the electronic resonator shall be obtained. We shall consider the resonators with symmetrical characteristics, to which the linear mechanical system with one degree of freedom and viscous friction adopted as a calculation model of the mechanical resonator, is related.

Then the module of the maximum relative value of the transform factor will be

$$K_{T_{rel}} = \sqrt{1 - \mu^2}, \qquad (3)$$

where $\mu = 4\nu_\omega / \Delta\omega_0^2$ is the dimensionless parameter, $\Delta\omega = \omega_0 / Q$ is the resonance band and Q is the $\boldsymbol{Q}$ – factor of the resonator.

To compare the parameters of the dynamic characteristic of the electronic resonator and the mechanical one, we shall express the μ parameter through the reduced number of oscillations

$$n_Q = \frac{\omega_0^2}{Q^2 v_\omega}, \quad \mu = 4/n_Q. \qquad (4)$$

The formula (3) can be presented in the form

$$K_{T_{rel}} = \sqrt{1 - 16/n_Q^2}. \qquad (5)$$

Further, we shall consider a linear mechanical system with one freedom degree consisting of a weight with the mass of m on the spring with the rigidity C and viscous friction damper with the damping coefficient β (Figure 1).

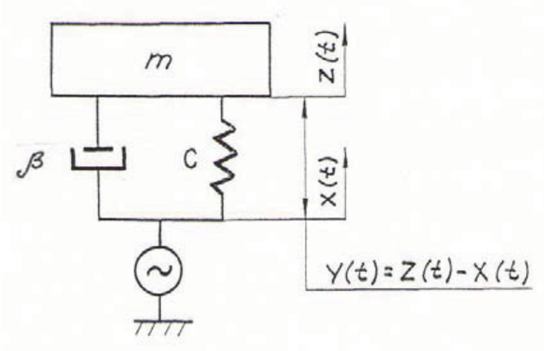

Figure 1 – Mechanical model of the resonator

When being subjected to the kinematic excitation (at the cost of motion of the supporting base), the differential equation of movement of the weight will look as follows

$$m \cdot \ddot{z} + \beta(\dot{z} - \dot{x}) + C(z - x) = 0, \qquad (6)$$

where z is the absolute displacement of the weight and x is the displacement of the base.

3. THE SOLUTION OF THE DIFFERENTIAL EQUATION OF NONLINEAR OSCILLATIONS

Let us introduce the angular frequency of the natural undamped oscillations of the system $\omega_0 = \sqrt{C/m}$ and the ratio $\beta/m = \omega_0/Q$, where Q is the quality factor of the mechanical system into the equation. After separation of the quantities dependent on x and z into the left and right side, the equation will look as follows

$$\ddot{z} + \frac{\omega_o}{Q}\dot{z} + \omega_o^2 z = \omega_o^2 x + \frac{\omega_o}{Q}\dot{x}. \qquad (7)$$

Now we shall transit from the equation of displacement to the differential equation of accelerations. Double differentiating the right and left sides of the equation, neglecting the initial phase of the oscillations and assuming the equation of the base in the complex form [1]

$$j_x = A_{jx} e^{i\varphi(t)}, \qquad (8)$$

will lead to the following equation

$$\frac{d^2 j_z}{dt^2} + \frac{\omega_0}{Q}\frac{dj_z}{dt} + \omega_0^2 j_z = \omega_0\left(\omega_0 + i\frac{\omega}{Q}\right) A_{jx} e^{i\varphi(t)}, \qquad (9)$$

where j_z is the acceleration of the mass.

The excitation frequency depends on the time

$$\omega = \dot{\varphi}(t), \qquad (10)$$

and the base acceleration amplitude is constant A_{jx}=const.

Taking into account the initial conditions $j_z = dj_z/dt = 0$ at $t = 0$, we shall get the absolute acceleration of the system

$$j_z = \frac{\omega_0}{\omega^*}\int_0^t \left(\omega_0 + i\frac{\omega(\tau)}{Q}\right) A_{jx} e^{\frac{\omega_0}{2Q}(\tau - t) + i\varphi(t)} \times$$

$$\times \sin\omega^*(t-\tau)d\tau \qquad (11)$$

where the natural frequency of the damped system:

$$\omega^* = \omega_0\sqrt{1 - 1/4Q^2}. \qquad (12)$$

To determine the amplitude of absolute acceleration of the system $A_{jz}=|j_z|$ at the specified accelerations of the base A_{jx} under the condition of non-stationary oscillations, we shall find the acceleration transfer factor of the system

$$K_j = \frac{A_{jz}}{A_{jx}} = \frac{1}{\sqrt{1 - \frac{1}{4Q^2}}}\left|\int_0^t \left(\omega_0 + i\frac{\omega(\tau)}{Q}\right) e^{\frac{\omega_0}{2Q}(\tau - t) + i\varphi(\tau)} \times\right.$$

$$\left.\times \sin\left[\omega_0\sqrt{1 - \frac{1}{4Q^2}}(t-\tau)\right]d\tau\right| \qquad (13)$$

The exponential and linear laws of scanning of the excitation frequency within the range $f_н$–$f_в$ for the practice of vibration tests of manufactured articles are of the highest interest. The exponential law is characterized by the constancy of the time of transit of the resonance band for all the manufactured articles to be tested

$$f_1 = f_H e^{\vartheta_{1f}\ln 2}, \qquad (14)$$

where the frequency scanning rate (octave/s) is equal to

$$\vartheta_{1f} = \frac{\ln(f_в/f_H)}{T\ln 2}, \qquad (15)$$

$f_н$ and $f_в$ are the lower and the upper frequency of the testing range, respectively, and T is the tine of single transit of the frequency range, s.

When the law is linear, the frequency scanning rate ϑ_{2f} (Hz/s) is constant over the whole frequency range

$$f_2 = f_н + \vartheta_{2f} t. \qquad (16)$$

The functions $\varphi(\tau)$ in the expression (13) will be obtained by integrating the dependencies (14) and (15).

From the real Re J and imaginary Im J parts of the integral (13)

$$\operatorname{Re} J = \int_0^t e^{\frac{\omega_0}{2Q}(\tau - t)} \sin\left[\omega_0 \sqrt{1 - \frac{1}{4Q^2}}(t - \tau)\right] \times$$

$$\times \left(-\frac{\omega(\tau)}{Q} \sin\varphi(\tau) + \omega_0 \cos\varphi(\tau) \right) d\tau \,, \tag{17}$$

$$\operatorname{Im} J = \int_0^t e^{\frac{\omega_0}{2Q}(\tau - t)} \sin\left[\omega_0 \sqrt{1 - \frac{1}{4Q^2}}(t - \tau)\right] \times$$

$$\times \left(\frac{\omega(\tau)}{Q} \cos\varphi(\tau) + \omega_0 \sin\varphi(\tau) \right) d\tau \,, \tag{18}$$

the acceleration transfer factor can be calculated:

$$K_j = \frac{\sqrt{(\operatorname{Re} J)^2 + (\operatorname{Im} J)^2}}{\sqrt{1 - 1/4Q^2}} \,. \tag{19}$$

Now we shall introduce the notion of the reduced number of oscillation which will be defined as a ratio of the exciting oscillations in the resonance band to the quality factor. For the exponential and linear laws of the frequency scanning rate, respectively, the calculation is performed using the following formulae

$$n_{1Q} = \frac{\omega_0}{Q^2 v_{1\omega} \ln 2}\,, \quad n_{2Q} = \frac{\omega_0^2}{Q^2 v_{2\omega}}\,, \tag{20}$$

where ϑ_ω is the frequency scanning rate.

The most important characteristic of the dynamic processes in case of transient oscillations is the maximum value of the acceleration transfer factor:

$$K_{jrel} = A_{jz\max} / A_{jx} \,. \tag{21}$$

To obtain the generalized dependence at different quality factors Q, it is convenient to operate with the relative K_{jrel} to be determined as the rate of K_{jmax} for transient oscillations to the value K_j=Q for stationary (steady-state) oscillations

$$K_{jrel} = K_{j\max} / Q \,. \tag{22}$$

4. RESULTS OF THEORETICAL AND EXPERIMENTAL RESEARCHES

The calculations were performed at different values of n_o and quality factor Q = 10, 20, 40, 80 and 160 for the exponential frequency scanning law and Q = 10, 20, 40 and 80 for the linear law. The values of Q, ω_H, ω_o, ϑ_ω were selected based on the condition $n_{1Q} = n_{2Q}$ and time of reaching the frequency ω_o being equal to one second. From the results of computation using a computer, the following values were found: t_{max} being the time of reaching the frequency ω_{max} at which the transfer factor has the maximum value K_{jmax} as well as t_1 and t_2 being the times of reaching the limiting frequencies ω_1 and ω_2 of the dynamic resonance band $\Delta\omega_d = \omega_2 - \omega_1$, which were determined with the values of $K_j = K_{j\max}/\sqrt{2}$ (Figure 2). Further, the mean frequency of the amplitude-frequency characteristics and the frequencies $\omega_{\max}$, ω_1, ω_2 were calculated using the formulae (14) and (16) with the found values of t_{max}, t_1, t_2 respectively

$$\varpi = \frac{1}{2}(\omega_1 + \omega_2) \,. \tag{23}$$

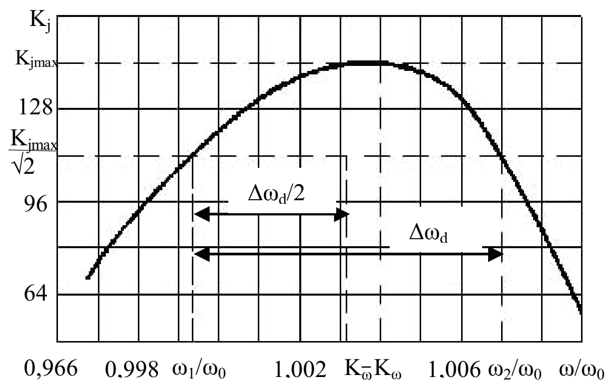

Figure 2 – Parameters of the dynamic

From the data processing results, the empirical dependences for finding the value of the relative acceleration transfer factor, coefficient of displacement of the frequency of the maximum of the dynamic AFC relatively to the frequency of the natural oscillations, coefficient of expansion of the dynamic resonance band and coefficient of displacement of the average AFC frequency relatively to that of the natural oscillations [2, 3] were obtained.

Figure 3 presents the AFC of the mechanical system with the linear law of scanning the frequency.

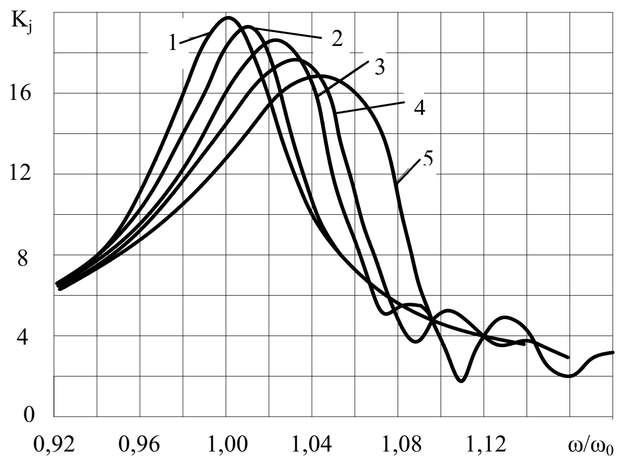

Figure 3 – AFC of the system: 1 – stationary AFC with the steady-state oscillations,
2 – dynamic AFC at n_{2Q}=12.96; 3 – the same at n_{2Q}=5.76; 4 – the same at n_{2Q}=4.00; 5 – the same at n_{2Q}=2.56

To calculate the relative acceleration transfer factor $\boldsymbol{K_{jrel}}$ of the mechanical resonator, the following formula is proposed (also see figure 4):

$$K_{j_{rel}} = (1 + \frac{0{,}708}{\sqrt{n_Q} e^{0{,}565\sqrt{n_Q}}})^{-1}. \quad (24)$$

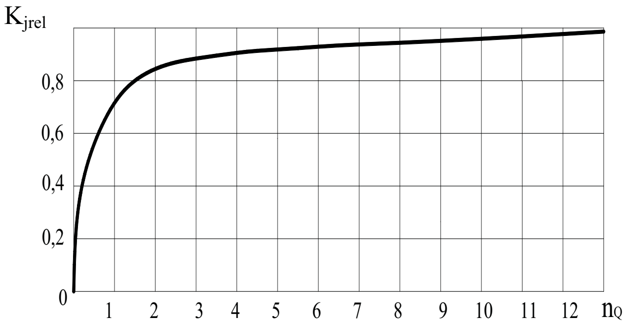

Figure 4 – The change of the maximum relative coefficient of the transfer to acceleration on the number of oscillations

The coefficient of expansion of the dynamic resonance band of the electronic resonator shall be determined from the formula

$$K_{\Delta\omega} = 1 + 1{,}25\mu^2 \quad (25)$$

and, with the account of the relationship (4), it will be

$$K_{\Delta\omega} = 1 + 20 / n_Q^2. \quad (26)$$

The coefficient of expansion of the band $K_{\Delta\omega}$ of the mechanical resonator for the exponential and linear frequency scanning laws, respectively, can be calculated from the formulae and showed on figure 5

$$K_{1\Delta\omega} = 1 + \frac{1{,}8 - 0{,}048Q + 3Q\sqrt{n_Q}}{Qn_Q(1 + 0{,}435\sqrt{n_Q})^2} \times$$

$$\times \frac{1}{1 + 0{,}0675(\sqrt{n_Q} - 2)^2}, \quad (27)$$

$$K_{2\Delta\omega} = 1 + \frac{1.8 - 0.048Q + 3Q\sqrt{n_{2Q}}}{Qn_{2Q}\left(1 + 0.435\sqrt{n_{2Q}}\right)} \times$$

$$\times \frac{1}{1 + 0.0675\left(\sqrt{n_{2Q}} - 2\right)^2}. \quad (28)$$

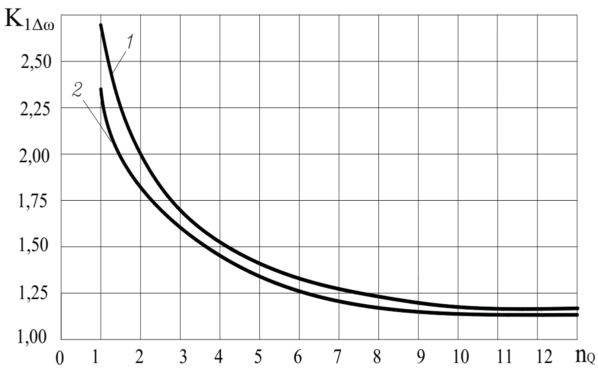

Figure 5 – The change of the coefficient of extend of resonant band of AFC with an exponential law of scanning frequency: 1 – Q=10; 2 – Q=160

Its value at $n_Q > 8$ is actually invariable relatively to the variation of the Q-factor.

The displacement of the maximum of the dynamic AFC of the electronic resonator

$$S = 2\mu \quad (29)$$

shall be expressed in the units of the generalized detuning:

$$x = \frac{2(\omega - \omega_0)Q}{\omega_0}. \quad (30)$$

Using the expressions (4), (29), (30), the coefficient of displacement of the maximum frequency can be calculated from the formula

$$K_\omega = \frac{\omega_{max}}{\omega_0} = 1 \pm \frac{4}{Qn_Q}. \quad (31)$$

The "+" sign corresponds to the increase of the excitation frequency and the "–" sign – to its decrease.

For the mechanical resonator, it is proposed to calculate K_ω from the formula (figure 6)

$$K_\omega = \frac{\omega_{max}}{\omega_0} = 1 \pm \frac{2{,}26}{Q\sqrt{n_Q}(1 + 0{,}394\sqrt{n_Q})}. \quad (32)$$

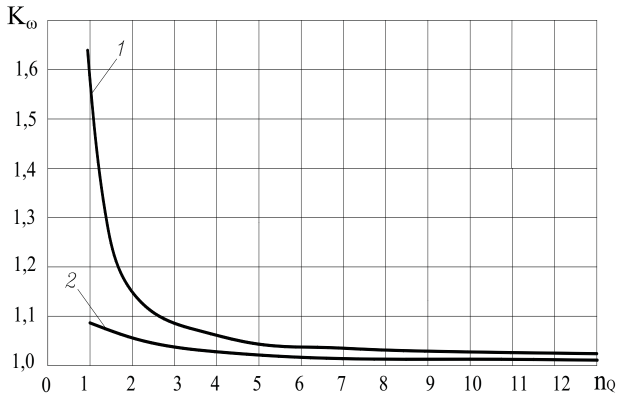

Figure 6 – The change of the coefficient of shift of AFC maximum frequency from the number of oscillations.

The centre of the dynamic resonance band is displaced relatively to the ω_0 frequency by the value

$$\mu = \frac{1}{2}(S_1 + S_2), \quad (33)$$

where S_1 and S_2 are the displacements of the limiting frequencies ω_1 and ω_2 of the band, respectively, expressed in the units of generalized detuning.

Using the expressions (4), (30), (32), the coefficient of displacement of the mean frequency of the dynamic band relatively to the frequency of the natural oscillations:

$$K_{\overline{\omega}} = \frac{\overline{\omega}}{\omega_0} = 1 \pm \frac{2}{Qn_Q}. \quad (34)$$

For calculating $K_{\overline{\omega}}$ a mechanical resonator, the following formula is proposed (also see figure 7):

$$K_{1\varpi} = \frac{\varpi}{\omega_0} = 1 \pm \frac{2{,}3}{Q\sqrt{n_{1Q}}\left(1 + 0.7\sqrt{n_{1Q}}\right)}, \quad (35)$$

$$K_{2\overline{\omega}} = 1 \pm \frac{2{,}19}{Q\sqrt{n_Q}\,(1 + 0{,}7\sqrt{n_Q})}. \quad (36)$$

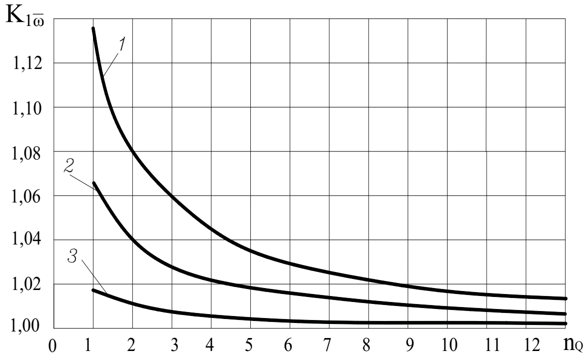

Figure 7 – The change of the coefficient of shift of middle frequencies AFC at the exponential law scanning from the number of oscillations: 1 – Q=10; 2 – Q=20; 3 – Q=80.

With the account of the assumptions adopted when calculating the integral (1), the range of application of the formulae (3), (25), (29), (33) shall be restricted to the value $\mu < 1/3$ ($n_Q > 12$), and these formulae are not applicable at all should μ be of the order of one. The error of these formulae is determined relatively to the parameters of the dynamic characteristic of the mechanical resonator.

The best coincidence with the data on the expansion of the resonance band and frequency displacement of the maximum is obtained at the values μ = 1.5–2 (n_Q = 2–2.67). The data on the cut of the maximum of the dynamic magnification factor coincide satisfactorily up to the values of $\mu < 0.5$ ($n_Q > 8$) only.

For the purpose of the additional checking of the trustworthiness of the results being obtained using the described technique, the problem of determining the amplitude of transient oscillations of the mechanical system was solved with the force excitation and linear frequency scanning law.

For a real resonator with the quality factor Q = 98 and resonance frequency of 316.9 Hz, the transient oscillations were registered on the storage oscilloscope (Figure 8). The resonator was excited within the range $f_в$ = 290–341 Hz. The frequency scanning rate was set to be ϑ_{2f} = 10, 50 and 100 Hz/s. The maximum amplitude of the resonator acceleration was measured in relative units at the constant amplitude of acceleration of the vibration stand table equal to 57 m/s^2. The acceleration amplitude was also determined with the steady-state resonance oscillations, when $f_в = f_г$ and $\vartheta_{2f} = 0$.

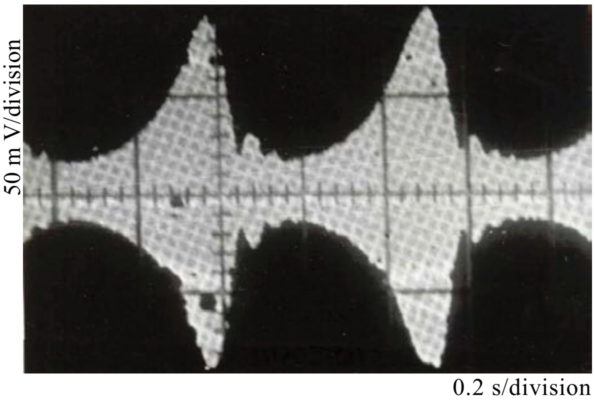

Figure 8 – Transient oscillations of the resonator at the frequency scanning rate ϑ_{2f}=100 Hz/s (n_{2Q}=1.31)

5. CONCLUSIONS

1. When the quality factor and the reduced number of exciting oscillations in the resonance band of the article are equal to one another, the value of displacement of the maximum of the dynamic AFC is independent on the frequency scanning law.
2. The maximum amplitude of the transient oscillations of the article for the same conditions does not vary as the scanning law does.
3. The selection of the lower portion of the testing range does not affect the parameters of the dynamic AFC when the equations mentioned in item 1 are true.
4. The frequency of displacement of the AFC maximum in case of transient oscillations is independent on the type of excitation (either force or kinematic one).
5. The frequency scanning law affects the value of the mean frequency of the AFC and width of the dynamic resonance band.
6. The proposed formulae describe the real dynamic processes with good degree of consistency.
7. The results of this paper can be used for designing the spectrum analyzers to be calculated based on the dynamic AFC parameters at arbitrarily high excitation frequency scanning rate, if the rate is constant.

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